

CHIRAL PERTURBATION THEORY, NON-LEPTONIC KAON DECAYS, AND THE LATTICE

MAARTEN GOLTERMAN

Department of Physics, Washington University, St. Louis MO 63130, USA
E-mail: maarten@aapje.wustl.edu

In this talk, I first motivate the use of Chiral Perturbation Theory in the context of Lattice QCD. In particular, I explain how partially quenched QCD, which has, in general, unequal valence- and sea-quark masses, can be used to obtain real-world (*i.e.* unquenched) results for low-energy constants. In the second part, I review how Chiral Perturbation Theory may be used to overcome theoretical difficulties which afflict the computation of non-leptonic kaon decay rates from Lattice QCD. I argue that it should be possible to determine at least the $O(p^2)$ weak low-energy constants reliably from numerical computations of the $K \rightarrow \pi$ and $K \rightarrow$ vacuum matrix elements of the corresponding weak operators.

1 Introduction

Ideally, Lattice QCD should lead to the unambiguous, first-principle determination of hadronic quantities. However, it does not work that way in practice, basically because of limits on available computational power. Because of these limits, approximations are needed which lead to systematic errors, and we need a theoretical understanding of these in order to obtain estimates of hadronic quantities with known and controlled errors. This talk addresses some of the limitations of Lattice QCD with respect to the physics of Goldstone Bosons (GBs), and explains how Chiral Perturbation Theory (ChPT) can be used to extract physical quantities from the often unphysical results of lattice computations.

The basic idea is that ChPT parametrizes QCD correlation functions involving GBs in terms of a number of parameters, the low-energy constants (LECs). To a given order in the chiral expansion, physical quantities are determined completely in terms of a finite number of LECs, and these LECs can then be obtained from (possibly other) correlation functions at often unphysical choices of masses and momenta which are more easily accessible to numerical computation. Such a program only works if the chiral expansion converges in the regions of interest (both physical and on the lattice), but this can in principle be checked by considering successive terms in the chiral expansion, once the LECs have been determined from fits to lattice results. Obviously, for such checks at least two orders in the chiral expansion need to be considered, and one-loop ChPT is essential.

A simple example is the pion mass, which is extracted from the appropriate euclidean two-point function. The computations are done in a finite volume L^3 , and, to avoid finite-size distortions, one has to require that $m_\pi L \gg 1$. Since, in practice, $L \approx 3$ fm is (currently) the largest attainable volume, demanding (say) $m_\pi L > 5$ leads to the requirement that $m_\pi > 330$ MeV on the lattice; in other words, the light quark masses have to be chosen unphysically large. But if ChPT converges in the range $300 \text{ MeV} < m_\pi < 700 \text{ MeV}$ or so, it can be used to determine the relevant LECs and extrapolate to the physical pion mass (*e.g.* to determine light quark masses). Once this works, there is actually no need to perform numerical computations in volumes large enough to contain a physical pion. Instead, additional computational power could for instance be used to go to smaller lattice spacings at the same physical volume, in order to reduce scaling violations.

In this talk, I will discuss two topics from this point of view. The first topic will be the use of “partially quenched” QCD (PQQCD), which extends the idea that, for a number of quantities of interest, simpler unphysical lattice computations may be all one needs. The second topic will be that of weak matrix elements for non-leptonic kaon decays. In this case, accessible euclidean correlation functions are “not physical” for a number of reasons, but ChPT can be used to extract relevant information.

2 Partially Quenched QCD

2.1 Why partially quenched?

In a lattice calculation, the quark fields are integrated over analytically, leading to a gluonic expectation value of the product of the fermion determinant and various quark propagators. For example,

$$\langle \bar{u} \gamma_5 d(x) \bar{d} \gamma_5 u(0) \rangle = \frac{\langle \gamma_5 (\not{D} + m_d)^{-1}(x, 0) \gamma_5 (\not{D} + m_u)^{-1}(0, x) \det(\not{D} + m) \rangle_{\text{glue}}}{\langle \det(\not{D} + m) \rangle_{\text{glue}}}. \quad (1)$$

It is clear that, in a computation, one could choose the quark mass in the determinant (the “sea” quark mass) unequal to those in the propagators (the “valence” quark masses), as indicated in eq. (1). This situation is referred to as “partial quenching.” The (fully) quenched theory is obtained by omitting the sea quarks altogether, or, equivalently, by sending $m = m_{\text{sea}} \rightarrow \infty$. A simple reason to do this is that it is much cheaper to compute a propagator than a determinant, so that one can vary the valence masses almost “for free,” unlike the sea-quark mass. This then raises two questions: is this useful for anything, and, can this be understood with field-theoretic methods?

To the first question, the answer is yes! For example, a partially quenched simulation with $N = 3$ degenerate light sea quarks (and arbitrary valence masses) gives information about the real world.¹ Consider again the pion mass. With N sea quarks of mass m_s , and two valence quarks of mass m_v , partially quenched ChPT (PQChPT; see next subsection) gives^{2,3}

$$m_\pi^2 = 2B(N)m_v \left(1 + \text{chiral logs} \right. \\ \left. + \frac{32B(N)}{f^2(N)} \left[(2L_8(N) - L_5(N))m_v + ((2L_6(N) - L_4(N))Nm_s) \right] \right), \quad (2)$$

where we follow the notation of Gasser and Leutwyler⁴ (but the pion decay constant $f(N)$ in the chiral limit is normalized such that it equals 132 MeV at m_π). We explicitly indicate the dependence on N . (The QCD dynamics (*i.e.* the determinant) and hence the LECs do not depend on the valence quarks.) Since the LECs depend *only* on N , it is clear that if we take $N = 3$, the values obtained from fitting this expression to lattice results are the real-world values. Note that only degenerate sea quarks are needed to get $2L_8 - L_5$ and $2L_6 - L_4$ separately. If we only use unquenched QCD, non-degenerate sea quarks would be needed to get the same information, or one would have to consider matrix elements like $\langle \pi | \bar{s}s | \pi \rangle$, which involve – numerically much harder – disconnected diagrams. It is clear that the $N = 3$ LECs cannot be obtained from quenched QCD, which has $N = 0$. If we can simulate PQQCD with three dynamical degenerate quarks at roughly the strange quark mass to a high enough accuracy to fit eq. (2), one could thus, for example, decide the fate of the Kaplan–Manohar ambiguity⁵ from first principles.

2.2 Partially quenched field theory

PQQCD can be defined as a path integral, by considering the lagrangian⁶

$$\mathcal{L} = \bar{q}_v^i (\not{D} + m_v^i) q_v^i + \bar{q}_s^\alpha (\not{D} + m_s^\alpha) q_s^\alpha + \bar{\tilde{q}}^j (\not{D} + m_v^j) \tilde{q}^j, \quad (3)$$

where q_v^i , $i = 1 \dots K$ are the valence quarks and q_s^α , $\alpha = 1 \dots N$ the sea quarks. The valence quarks are made into such by adding K “ghost” quarks \tilde{q}^j , $j = 1 \dots K$, which are identical to the valence quarks q_v^i (with the same masses m_v^i), except that they are given opposite statistics.⁷ Therefore, the valence and ghost determinants cancel, leaving only the sea-quark determinant, as in eq. (1). Obviously, the theory defined in this way is not a “healthy” field theory (it violates spin-statistics), but the interesting point is that one can

study its unphysical, euclidean correlation functions, and extract information about the real world, as in the example of the previous subsection.

For this, we need to adapt ChPT to the partially quenched situation. This can be done systematically by observing that \mathcal{L} has a large chiral symmetry group, $SU(N+K|K)_L \times SU(N+K|K)_R$, where $SU(N+K|K)$ is a graded version of $SU(N+2K)$ because it transforms fermions (q_v, q_s) into bosons (\tilde{q}), and *vice versa*. Based on this, PQChPT can be developed in much the same way as in the usual case, using supertraces and superdeterminants to build graded-group invariants.^{8,6} Quenched QCD corresponds to $N=0$. Note that partially quenched QCD, unlike quenched QCD, contains QCD (for which $N=3, K=0$) as a special case.⁶

I have not enough space to review (P)QChPT,⁹ and will limit myself to a brief discussion of neutral mesons (*i.e.* those containing quarks and anti-quarks of the same flavor). The tree-level two-point function for mesons $\bar{q}^i \gamma_5 q^i$ (no sum over i) is,^{6,2,1,10} in the limit of large η' mass, equal to

$$\delta_{ij} \frac{1}{p^2 + M_{VV}^2} - \frac{1}{N} \frac{1}{p^2 + M_{VV}^2} - \frac{1}{N} \frac{M_{SS}^2 - M_{VV}^2}{(p^2 + M_{VV}^2)^2}, \quad (4)$$

where M_{VV} (M_{SS}) is the mass of a meson made from (degenerate) valence (sea) quarks. In the quenched theory ($N=0$), the η' cannot be decoupled, leading to a two-point function^{8,11}

$$\delta_{ij} \frac{1}{p^2 + M_{VV}^2} - \frac{m_0^2/3}{(p^2 + M_{VV}^2)^2}, \quad (5)$$

where m_0 is the parameter corresponding to the singlet part of the η' mass in the unquenched theory. The remarkable new feature in these two-point functions is the double-pole term. (Note that it disappears for $M_{VV} = M_{SS}$, $N \neq 0$, as it should.) This double pole leads to several infrared diseases, which are a consequence of (partial) quenching, and not present in full QCD:

- “Enhanced chiral logarithms.”^{8,6,11} Instead of having the usual chiral logs like $M^2 \log M^2$, one finds logs such as $(M_{SS}^2 - M_{VV}^2) \log M_{VV}^2$ ($N \neq 0$) or $m_0^2 \log M_{VV}^2$ ($N=0$), which diverge in the chiral limit $M_{VV} \rightarrow 0$.
- “Enhanced finite-volume effects.”^{12,13} See subsection 3.2.

2.3 Quenched ChPT and Lattice QCD

An important question is whether numerical computations are precise enough to discern one-loop effects in ChPT. The most obvious place to look is the quenched numerical results for GB masses and decay constants, for which the

most extensive and statistically significant numerical results are available. In quenched ChPT, one finds,^{8,11} for degenerate quarks

$$\frac{m_\pi^2}{8\pi^2 f^2} = y \left(1 - \left(\delta - \frac{2}{3} \alpha y \right) \log y - \delta + \frac{1}{3} \alpha y + 8(2\lambda_8 - \lambda_5)y \right), \quad (6)$$

$$f_\pi/f = 1 + 4\lambda_5 y,$$

where $\lambda_i = (4\pi)^2 L_i$ are the (quenched) $O(p^4)$ LECs, and

$$y \equiv \frac{2Bm}{8\pi^2 f^2}, \quad \delta \equiv \frac{m_0^2/3}{8\pi^2 f^2}. \quad (7)$$

(There are similar expressions for non-degenerate quark masses.) The parameter α parametrizes momentum dependence of the singlet GB two-point vertex, and appears in the renormalization of the η' field.^{8,4} There are really two questions: 1) Given certain numerical results, can we see the chiral logs? and 2) Can we determine $\lambda_{5,8}$?

CP-PACS¹⁴ has high statistics results at quark masses ranging from $m_\pi \sim 300$ to 750 MeV, extrapolated to the continuum limit, and shows that this is sufficient to fit δ from eq. (6), setting $\alpha = 0$. With this restriction, they see the chiral logs, and find $\delta \approx 0.1$. This answers the first question. A full analysis of their results has not appeared yet, but estimates for $\lambda_{5,8}$ can also be extracted. From their fits, I find, for example, $10^3(2L_8 - L_5) \approx 0.3$.¹⁵ With these values, eq. (6) appears to converge well. These numbers were recently confirmed by Duncan *et al.*,¹⁶ who find $\delta = 0.065(13)$ (at one value of the lattice spacing, with improved Wilson fermions), $10^3 L_5 = 2.5(5)$ (*cf.* the value of the Alpha collaboration below!) and $10^3(2L_8 - L_5) = 0.4(2)$ (again, taking $\alpha = 0$). It remains to be seen what can be said when α is not constrained to vanish.

The Alpha collaboration¹⁷ has recently presented high-statistics, continuum-extrapolated results in the range 590 – 670 MeV, which, as they point out, is not sufficient to demonstrate the presence of chiral logs. However, using eq. (6) with the CP-PACS value of $\delta = 0.12(2)$ (assuming that the one-loop chiral expansion converges for this mass range), they find, for $\alpha = 0$

$$10^3 L_5 = 0.78 \pm 0.05 \pm 0.20, \quad 10^3(2L_8 - L_5) = 0.28 \pm 0.04 \pm 0.10 \pm 0.06, \quad (8)$$

where the 1st (2nd, 3rd) error is statistical (higher-order uncertainty, error on δ). Arguing that, if $\alpha = 0.5$, $\delta = 0.05(2)$ would be reasonable, they find

$$10^3(2L_8 - L_5) = 0.02, \quad (9)$$

with similar errors. It is clear that this value is very sensitive to α . All these estimates are quenched, and should not be directly compared to the real world. (The quenched LECs considered here do not even run.) It should

also be noted that in partially quenched simulations, if the quark masses are small enough, the η' decouples, and the parameters δ and α do not appear in the ChPT expressions¹⁸ — the coefficients in the logs are fully determined by chiral symmetry. The logarithm in eq. (6) is a quenching artifact. (However, δ and α do appear in PQChPT if the quark masses are not small enough.³)

3 Non-leptonic two-body kaon decays

3.1 $\Delta I = 3/2$

The simplest non-leptonic kaon decay is $K^+ \rightarrow \pi^+\pi^0$, which is pure $\Delta I = 3/2$. In order to determine this decay rate, one needs the weak matrix element

$$\langle \pi^+\pi^0 | \bar{s}_L \gamma_\mu (d_L \bar{u}_L \gamma_\mu u_L - u_L \bar{d}_L \gamma_\mu d_L + u_L \bar{u}_L \gamma_\mu d_L) | K^+ \rangle, \quad (10)$$

where u_L denotes the left-handed part of u , *etc.* On the lattice, one computes euclidean correlation functions, and this matrix element is contained in

$$C(t_2, t_1) = \langle 0 | \pi^+(t_2) \pi^0(t_2) O_{\text{weak}}(t_1) K^-(0) | 0 \rangle, \quad (11)$$

where $\pi^0(t_2)$ is a pion field with arbitrary momentum \vec{q} at time t_2 , *etc.* (we take the kaon to be at rest). Inserting complete sets of states on both sides of O_{weak} , and keeping only the leading exponential in t_1 on the kaon side, we find (with $t_2 > t_1 > 0$)

$$C(t_2, t_1) \sim \sum_n \langle \pi^+(-\vec{q}) \pi^0(\vec{q}) | n \rangle \langle n | O_{\text{weak}} | K^+ \rangle \langle K^+ | K^- | 0 \rangle e^{-E_n(t_2-t_1) - m_K t_1}. \quad (12)$$

The leading exponential corresponds to the state $|n\rangle$ with the lowest energy, and this is the state with both pions at rest (*i.e.* $\vec{q} = 0$), with $E = 2m_\pi$! This is a manifestation of the so-called Maiani–Testa theorem,¹⁹ which states that it is impossible to obtain the physical matrix element from the large-time behavior of the euclidean correlation function when there are more particles in the final state. All we get is the *unphysical* matrix element with all mesons at rest. Note that this conclusion does not depend on what we pick \vec{q} to be: only the total momentum in the decay is conserved. The desired matrix element, for which $|\vec{q}| = \frac{1}{2}\sqrt{m_K^2 - 4m_\pi^2}$, is buried in the excited states. The insertion of O_{weak} at a fixed time does not conserve energy.

At present, there are two approaches to this problem. One approach is to make use of the fact that the desired physical matrix element can be obtained from $C(t_2, t_1)$ in a finite spatial volume L^3 , in which the excited energy levels are discrete.²⁰ With periodic boundary conditions, the energies are quantized

roughly as $E_{\vec{n}} = 2\sqrt{m_\pi^2 + (2\pi\vec{n})^2/L^2}$ (plus $O(1/L^3)$ corrections due to final-state interactions (FSI)), and one finds that the first excited states $|\vec{n}| = 1$ have energy equal to the incoming energy m_K if $m_\pi L \approx 4$ for realistic meson masses. So, if it is possible to determine the first excited-state exponential in $C(t_2, t_1)$ reliably on the lattice, one obtains the physical matrix element in a finite (but rather large) volume. This matrix element can be then converted to infinite volume, with a correction factor rather close to one.²⁰

This method, while theoretically “clean,” may turn out to be hard to implement in practice. First, one needs numerical computations with a high enough precision to extract the first excited state. Then, one needs $2m_\pi < m_K < 4m_\pi$ (the latter inequality such that the rescattering of the pions is elastic), and this implies rather small up and down quark masses (of order $m_{\text{strange}}/8$ or less) if $m_K \approx 500$ MeV. For light quark masses at $\sim m_{\text{strange}}/8$ this means $L \approx 5$ fermi, which is large by present standards. With a larger lattice kaon mass, L can be smaller. ChPT can be used to extrapolate to physical masses.²¹ Because of the difficulty of the required numerical computations, one would like to test these ideas in a (partially) quenched setting, but it is not clear at the moment what the effects of quenching would be in this approach. (For instance, “enhanced” finite volume effects may occur,^{12,13} see below.) It would be interesting to investigate this in (P)QChPT.

An alternative, and in fact older, idea is to compute the unphysical matrix element with all mesons at rest from the dominant term in eq. (12), for which $|n\rangle = |\pi^+(\vec{0})\pi^0(\vec{0})\rangle$,²² and use ChPT in order to convert the result to the physical matrix element. ChPT thus “corrects for” the systematic error from using unphysical (energy non-conserving) momenta, and unphysical quark masses. In addition, ChPT can also be used to estimate effects from finite-volume, excited states and (partial) quenching. Whether this works in practice depends on how well the chiral expansion converges for the situation at hand. For the physical amplitude, one finds, for $\Delta I = 3/2$ (in infinite volume),^{21,23}

$$\langle \pi^+\pi^0 | O_{\text{weak}} | K^+ \rangle_{\text{p}} = -\frac{4i\alpha^{27}}{f^3}(m_K^2 - m_\pi^2) \left(1 + \text{chiral logs}(\Lambda) + d(\Lambda) \frac{m_K^2}{(4\pi f)^2} \right), \quad (13)$$

where α^{27} is the $O(p^2)$ $\Delta I = 3/2$ LEC, and $d(\Lambda)$ is a linear combination of $O(p^4)$ LECs, at the scale Λ (we ignore an $O(p^4)$ term proportional to m_π^2). f is the pion-decay constant in the chiral limit. For physical pion and kaon masses, and at a scale $\Lambda \sim m_\rho$ to 1 GeV, the chiral logs are rather small. Phenomenological estimates of α^{27} and $d(\Lambda)$ exist.²⁴ For the unphysical matrix element, assuming degenerate valence quark masses corresponding to

a valence meson mass M_π , one finds^{23,3}

$$\begin{aligned} \langle \pi^+ \pi^0 | O_{\text{weak}} | K^+ \rangle_{\text{u}} = & - \left(\frac{4i\alpha^{27}}{f^3} \right)_{\text{PQ}} 2M_\pi^2 \left(1 - N \frac{M_{VS}^2}{(4\pi f)^2} \left[\log \frac{M_{VS}^2}{\Lambda^2} + d'(\Lambda) \right] \right. \\ & \left. + \frac{M_\pi^2}{(4\pi f)^2} \left[-3 \log \frac{M_\pi^2}{\Lambda^2} + d''(\Lambda) + \frac{17.8}{M_\pi L} + \frac{12\pi^2}{(M_\pi L)^3} \right] \right). \quad (14) \end{aligned}$$

Here N is the number of sea quarks, M_{VS} is the mass of a meson made out of a valence and a sea quark, and d' , d'' are $O(p^4)$ LECs of the (partially) quenched theory. Note that α^{27} , f and other LECs are different for different values of N , and only equal to those of the real world for $N = 3$. This result also holds for the quenched case, $N = 0$.

The finite-volume effects are power-like (we ignored exponentially small corrections), and potentially large. For example, with $f = 160$ MeV, $M_\pi = 500$ MeV and $M_\pi L = 6$, this correction is about 22% of tree level. They arise from the ChPT one-loop diagram in which the final-state pions of the weak decay vertex rescatter. In hamiltonian perturbation theory this corresponds to a contribution due to intermediate pions with a momentum $\vec{k} = 2\pi\vec{n}/L$, leading to an energy denominator

$$\begin{aligned} \frac{1}{L^3} \sum_{\vec{k} \neq 0} \frac{1}{\sqrt{M_\pi^2 + \vec{k}^2} - M_\pi} &= \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\sqrt{M_\pi^2 + \vec{k}^2} - M_\pi} \\ &+ \frac{c}{M_\pi L^3} (M_\pi L)^2 \left(1 + O\left(\frac{1}{(M_\pi L)^2}\right) \right), \quad (15) \end{aligned}$$

where c is a numerical constant (for a more extensive discussion, see Bernard and Golterman¹² and refs. therein).

The combined one-loop corrections in eq. (14) are rather large for typical lattice values of the parameters, and tend to increase the size of the matrix element.^{23,3} The most recent numerical computation of this matrix element was done by JLQCD,²⁵ in the quenched approximation (*i.e.* $N = 0$), at an inverse lattice spacing of 2 GeV. It was found that eq. (14) describes the numerical results reasonably well, in particular the volume dependence, which has no scale ambiguities. They obtained $d''(m_\rho)/(4\pi)^2 \approx 0.025$ and $d''(1 \text{ GeV})/(4\pi)^2 \approx 0.015$, with the difference in agreement with eq. (14).

A variant of this method is to take, on the lattice, $m_K = 2m_\pi$.²⁶ In this case the matrix element with all mesons at rest conserves energy, and, in that sense, is physical. It is also closer to the real world than the case with $m_K = m_\pi$, the choice made by JLQCD and in earlier lattice simulations. However, chiral corrections are typically still large.²⁷

3.2 $\Delta I = 1/2$

The matrix element for $\Delta I = 1/2$ non-leptonic kaon decay constitutes a more difficult case for many reasons. First, lattice computations are harder because of the so-called “eye” diagrams (containing quark propagators starting and ending at the same space-time point). Also, unlike the $\Delta I = 3/2$ case, mixing with the lower-dimension operator $(m_s - m_d)\bar{s}\gamma_5 d$ occurs. This problem can be avoided by choosing degenerate (valence) quarks,²² or, in the case that $m_K = 2m_\pi$, by using an improved action.²⁶ In practice, we are therefore limited to the computation of the matrix element with $m_K = m_\pi$ or $m_K = 2m_\pi$ and all mesons at rest, if we can only determine the leading large-time behavior of eq. (11).

This means that ChPT will be needed in order to extract the relevant LECs from the unphysical matrix element, in order to calculate the physical matrix element. The question arises again which order in ChPT will be needed in practice. At one loop a problem occurs (for both $\Delta I = 1/2, 3/2$), as it turns out that the $O(p^4)$ LECs needed for the physical matrix element cannot be determined from the unphysical matrix element. This can be seen as follows. With arbitrary external four-momenta $p_K = (im_K, \vec{0})$, p_{π_1} and p_{π_2} with $p_{\pi_1}^2 = p_{\pi_2}^2 = -m_\pi^2$, the contribution from $O(p^4)$ operators is expected to be of the form

$$\frac{1}{(4\pi f)^2} [Am_K^2 + Bm_\pi^2 + 2Cp_{\pi_1}p_{\pi_2} + D(p_Kp_{\pi_1} + p_Kp_{\pi_2})] \times \text{tree level}, \quad (16)$$

where A , B , C and D are independent (linear combinations of) $O(p^4)$ LECs. For the physical matrix element we have $p_K = p_{\pi_1} + p_{\pi_2}$, and the polynomial in square brackets in eq. (16) reduces to $[(A - C - D)m_K^2 + (B + 2C)m_\pi^2]$. For the unphysical matrix element we have $p_{\pi_1} = p_{\pi_2} = (im_\pi, \vec{0})$, and the polynomial becomes $[(A + B - 2C - 2D)m_\pi^2]$ for $m_K = m_\pi$, and $[(4A + B - 2C - 4D)m_\pi^2]$ for $m_K = 2m_\pi$. In neither case do we obtain the desired combinations of $O(p^4)$ LECs from the unphysical matrix element. In fact, in the mass-degenerate case, new, total-derivative $O(p^4)$ operators contribute because in this case energy is not conserved. At $O(p^2)$ this does not happen because of CPS symmetry (CP followed by $d \leftrightarrow s$, $m_d \leftrightarrow m_s$),²⁸ but CPS symmetry is not sufficient at $O(p^4)$. All this shows that the unphysical matrix elements do not contain enough information at $O(p^4)$.

Last but not least, there are new problems associated with (partial) quenching. Even in the mass-degenerate case, the $\Delta I = 1/2$ operator couples to the η' , and therefore the matrix element is afflicted by the double pole in the η' two-point function (eqs. (4,5)). (If $SU(3)$ is unbroken, the η' does not couple to the $\Delta I = 3/2$ operator.) It turns out that enhanced chiral logs are

either absent¹³ ($\Delta I = 1/2$ at $m_K = m_\pi$) or numerically small²⁷ ($\Delta I = 3/2$ at $m_K = 2m_\pi$), at one loop in ChPT. However, in the $\Delta I = 1/2$ case, enhanced finite volume effects occur at one loop,¹³ giving rise to “corrections” of order L to the matrix element! The double pole can occur on the internal lines of the meson rescattering diagram, leading to terms proportional to (*cf.* eq. (15))

$$\frac{1}{L^3} \sum_{\vec{k} \neq 0} \frac{1}{\left(\sqrt{M_\pi^2 + \vec{k}^2} - M_\pi \right)^2}. \quad (17)$$

In finite volume, the smallest momentum has $|\vec{k}| = 2\pi/L$, leading to contributions of order $(1/L^3) \times L^4 = L$. This is another manifestation of the infrared diseases afflicting the (partially) quenched theory. In the quenched theory ($N = 0$), enhanced finite-volume effects are proportional to m_0^2 (*cf.* eq. (5)), while in the partially quenched case ($N \neq 0$) they are proportional to $M_{SS}^2 - M_{VV}^2$ (*cf.* eq. (4)). (For a more detailed version of this argument, see Bernard and Golterman.¹²)

All this means that we may have to be more modest, at least at present. While we may not easily acquire sufficient information to determine non-leptonic decay rates to $O(p^4)$, it would be already interesting to determine the $O(p^2)$ weak LEC α^{27} and the corresponding octet LEC α_1^8 from the lattice. This, however, can be accomplished using simpler weak matrix elements, which avoid the enhanced finite-volume complication described above.

3.3 $K \rightarrow \pi\pi$ from $K \rightarrow \pi$ and $K \rightarrow 0$

An easy way to explain this idea²⁸ is by working at tree level in ChPT first. The physical matrix elements of interest are

$$\begin{aligned} \langle \pi\pi | O_{\text{octet}} | K \rangle &= \frac{4i}{f^3} (m_K^2 - m_\pi^2) \alpha_1^8, \\ \langle \pi\pi | O_{27\text{-plet}} | K \rangle &= -\frac{4i}{f^3} (m_K^2 - m_\pi^2) \alpha^{27}, \quad (\Delta I = \frac{1}{2}, \frac{3}{2}), \end{aligned} \quad (18)$$

where α^{27} and α_1^8 are the $O(p^2)$ weak LECs. However, we can also consider the unphysical, but simpler matrix elements

$$\begin{aligned} \langle \pi | O_{\text{octet}} | K \rangle &= \frac{4}{f^2} M^2 (\alpha_1^8 - \alpha_2^8), \\ \langle \pi | O_{27\text{-plet}} | K \rangle &= -\frac{4}{f^2} M^2 \alpha^{27}, \quad (\Delta I = \frac{1}{2}, \frac{3}{2}), \\ \langle 0 | O_{\text{octet}} | K \rangle &= \frac{4i}{f} (m_K^2 - m_\pi^2) \alpha_2^8, \end{aligned} \quad (19)$$

where in the $K \rightarrow \pi$ case we choose $m_K = m_\pi = M$. α_2^8 is the $O(p^2)$ LEC associated with the weak mass term.^{28,21,10} There are no FSI, and therefore none of the unphysical effects on the lattice associated with them.^a One-loop corrections to eq. (19) have been calculated in (P)QChPT including all contributions from $O(p^4)$,^{10,13} and are typically found to be large (assuming a natural cutoff and no sizable cancellations between chiral logs and $O(p^4)$ contact terms). A number of lattice collaborations are presently pursuing this approach, and it should be interesting to see what can be learned by fitting numerical results to $O(p^4)$ ChPT. If, for certain results from the lattice, the convergence of ChPT is reasonable, one obtains the $O(p^2)$ LECs α^{27} and α_1^8 . Even though that does only give kaon decay rates to lowest order in ChPT, lattice results for these LECs are interesting by themselves, and can be compared with phenomenological estimates.²⁴ Of course one would also obtain information about certain $O(p^4)$ LECs from such fits, but, as in the case of unphysical $K \rightarrow \pi\pi$ matrix elements, one does not get enough information to determine all $O(p^4)$ LECs relevant for the *physical* matrix elements.¹⁰ Results for α^{27} and α_1^8 from $K \rightarrow \pi$ and $K \rightarrow 0$ can of course be checked against those obtained from unphysical $K \rightarrow \pi\pi$ matrix elements, in principle.

4 Conclusion

In this talk, I have argued that ChPT plays a crucial role in extracting physics from numerical computations using Lattice QCD. The generic setting of such computations is that of partially quenched QCD, and ChPT can be systematically adapted to this situation. In fact, PQQCD with three light quarks subsumes the real world, *i.e.* unquenched QCD with valence-quark masses equal to those of the sea quarks. It follows that LECs of the partially quenched theory are the same as those of the real world. This means that they can often be determined from computations using unequal (and thus unphysical) values of the sea- and valence-quark masses, allowing to pick choices which are accessible with presently available computer resources (*cf.* section 2.1).

In contrast, quenched QCD has no sea quarks, and is therefore different from QCD. Because it is relatively cheap to simulate, it is a useful tool for investigating the issues discussed in this talk, for instance the convergence of ChPT at realistic values of the parameters.

In the second part, I addressed non-leptonic kaon decays. These involve the computation of weak matrix elements with more than one strongly inter-

^aAnother issue is whether ChPT at $O(p^4)$ takes *physical* FSI reliably into account. See Pallante and Pich²⁹ and refs. therein. Their results for FSI corrections to α_1^8 appear to be in reasonable agreement with the one-loop corrections of ChPT.²⁴

acting particle in the final state, and this leads to several difficulties. The first difficulty is that, when final-state interactions are present, one does not easily obtain matrix elements for a physical choice of momenta. This difficulty originates in the fact that on the lattice we only have access to the large-time regime of euclidean correlation functions. The second difficulty is that (partial) quenching artifacts, due to the unphysical infrared behavior of (partially) quenched theories, are more severe in this case. However, ChPT provides an analytic framework for parametrizing all these unphysical effects in terms of the LECs of the physical theory (when three reasonably light sea quarks are used on the lattice), and therefore can be used to extract phenomenologically relevant information from unphysical matrix elements. In particular, I argued that it should be possible to determine the leading weak LECs α^{27} and α_1^8 reliably from Lattice QCD.

Acknowledgments

I would like to thank the organizers for giving me the opportunity to present this (selective) overview, and Tomoteru Yoshie for making results from the CP-PACS collaboration available before publication. I thank Claude Bernard, Ka Chun Leung, Elisabetta Pallante, Steve Sharpe, Noam Shores, as well as many members of the CP-PACS collaboration for discussions. I thank Elisabetta Pallante and Steve Sharpe for comments on this manuscript. This work is supported in part by the US Dept. of Energy.

References

1. S. Sharpe, N. Shores, Nucl. Phys. B (Proc. Suppl.) 83-84 (2000) 968; hep-lat/0006017
2. S. Sharpe, Phys. Rev. D56 (1997) 7052
3. M. Golterman, K.C. Leung, Phys. Rev. D57 (1998) 5703
4. J. Gasser, H. Leutwyler, Nucl. Phys. B250 (1985) 465
5. D. Kaplan, A. Manohar, Phys. Rev. Lett. 56 (1986) 2004; A. Cohen, D. Kaplan, A. Nelson, JHEP 9911 (1999) 027
6. C. Bernard, M. Golterman, Phys. Rev. D49 (1994) 486
7. A. Morel, J. Physique 48 (1987) 1111
8. C. Bernard, M. Golterman, Phys. Rev. D46 (1992) 853
9. M. Golterman, Acta Phys. Polon. B25 (1994) 1731
10. M. Golterman, E. Pallante, JHEP 0008 (2000) 023
11. S. Sharpe, Phys. Rev. D46 (1992) 3146
12. C. Bernard, M. Golterman, Phys. Rev. D53 (1996) 476

13. M. Golterman, E. Pallante, Nucl. Phys. B (Proc. Suppl.) 83-84 (2000) 250
14. CP-PACS collaboration (S. Aoki *et al.*), Phys. Rev. Lett. 84 (2000) 238; Nucl. Phys. (Proc. Suppl.) 73 (1999) 189
15. T. Yoshie, private communication
16. A. Duncan *et al.*, hep-lat/0007010
17. J. Heitger, R. Sommer, H. Wittig, hep-lat/0006026
18. S. Sharpe, Y. Zhang, Phys. Rev. D53 (1996) 5125
19. L. Maiani, M. Testa, Phys. Lett. B245 (1990) 585
20. L. Lellouch, M. Lüscher, hep-lat/0003023
21. see *e.g.* J. Bijnens, E. Pallante, J. Prades, Nucl. Phys. B521 (1998) 305
22. C. Bernard *et al.*, Nucl. Phys. B (Proc. Suppl.) 4 (1988) 483
23. M. Golterman, K.C. Leung, Phys. Rev. D56 (1997) 2950; E. Pallante, JHEP 9901 (1999) 012
24. J. Kambor, J. Missimer, D. Wyler, Phys. Lett. B261 (1991) 496
25. JLQCD collaboration (S. Aoki *et al.*), Phys.Rev. D58 (1998) 054503
26. C. Dawson *et al.*, Nucl. Phys. B514 (1998) 313
27. M. Golterman, K.C.Leung, Phys. Rev. D58 (1998) 097503
28. C. Bernard *et al.*, Phys. Rev. D32 (1985) 2343
29. E. Pallante, A. Pich, Phys. Rev. Lett. 84 (2000) 2568; hep-ph/0007208